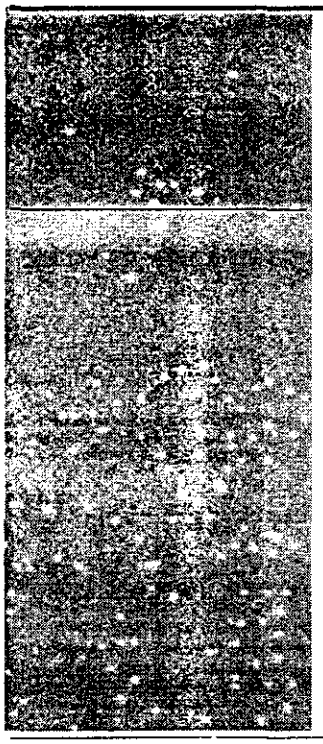
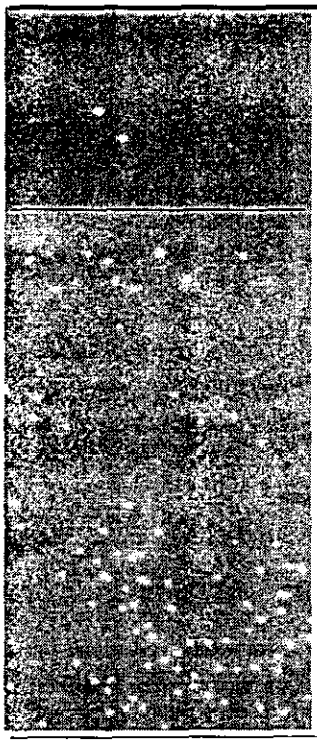
NEUTRONIC ANALYSIS OF REACTORS

Jean Koclas



About the Author

THE AUTHOR is currently professor of nuclear engineering at Ecole Polytechnique de Montréal. He is a graduate of McGill University, of Ecole Polytechnique de Montréal, and of the Massachusetts Institute of Technology (Ph.D., 1980). He worked at the Chalk River Laboratories (AECL) as a research scientist, and at the Gentilly-2 NGS of Hydro-Québec before he joined the Nuclear Engineering Institute of the Mechanical Engineering Department in 1992. Most of his research interests are in the areas of nuclear reactor simulation and space-time kinetics.



Foreword

THE MATERIAL PRESENTED here are the notes prepared by the author for a series of advanced lectures given both at Chulalongkorn University, Bangkok, Thailand, and at Ecole Polytechnique de Montréal, Montréal, Canada, in the winter months of 1998.

The subject matter covers a broad spectrum of the methods used to solve reactor physics problems in both the steady state and the non-stationary cases. Most of the emphasis is put on the methods of space-time kinetics, although very specific aspects of reactor statics are discussed.

Using the contents of these notes as a starting point, much of the current scientific literature on reactor physics should be within grasp of the reader. Also, the structure, and much of the technical contents, of many

of the modern computer codes used for complex reactor physics simulations become much clearer.

The background material required includes basic knowledge of reactor physics, linear algebra, and numerical analysis. The mathematics are derived with much of the intermediate steps included, so that it is easier for the reader to go through them.

Jean Koclas

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